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The inventory model for single vendor and multiple buyers with different deteriorations

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Abstract
This paper develops an integrated inventory model consisting of single-vendor and multiple-buyer system. Here we consider two types of buyers. The First type of buyers is using two warehouse inventory system with different deterioration and price discount is allowed for all items. The second type of buyers is returning the unsold items after certain time.The demand function is linear. This model provides an optimal solution for the total annual cost of the vendor and the buyer. The numerical example is provided to illustrate the model.

Keywords: two warehouse, vendor-buyer, price discount, different deterioration.

1. Introduction

In the existing literature, most of the inventory model derives from the buyer's point of view. This optimal decision policy may not be advantageous in economic term for the vendor. Thus, there is need to derive a joint policy which turns out to be a win-win strategy for both. Most of the research article is derived from independent policy which is either beneficial to vendor or buyer, but there are different views for different players and maybe they did not accept it globally. Ina competitive global market, an integrated policy should be developed when a decision is to be made which is favorable to both the players. Also, the integrated inventory policy minimizes the overall integrated cost of the entire system. [3]Clark and Scarf (1970) studied the vendor-buyer integration for the first time. [2]Banerjee (1986) extended Clark and Scarf model by introducing finite replenishment rate. [4]Goyal(1988) extended Banajee's model by relaxing the assumptions of the lot-for-lot production.[7]Nita H.Shah et al.2010 discussed joint vendor and buyer inventory strategy for deteriorating items with salvage value and also extended single vendor and multiple buyer on demand declining market. [8]Nita H.Shah et al.2013 derived joint optimal policy for a variable deteriorating inventory system of vendor buyer.
Two-warehouse inventory concept for merchants is an age-old one. This is followed to avoid frequent transportation inconvenience, to avail the advantage of price concession, to guard the scarcity of the commodity, etc. Now-a-days, the two warehouse inventory system has become more important due to prevailing volatile marketing condition and stiff competitions among the gradually increasing national and international sellers. Some research has already been performed in this field of study. [5]Hartely (1976) was the first to consider the effects of a two-warehouse system model in an inventory model with an RW storage policy. [10]Sarma (1983) developed a two-warehouse with constant demand. [9]Pakkala and Achary (1994) conferred a two level storage inventory model for deteriorating things with bulk unharness rule. [12]Yang (2004) extended partial backlogging then compared the two-warehouse model supported the minimum price

approach. [11]Wee et al.(2005) thought of a two-warehouse model with constant demand and weibull distribution deterioration under inflation.[1]Ajay singh Yadav and Anupam swami(2013)extended effect of permissible delay on two-warehouse inventory model for deteriorating itemswith shortages. Jasvinder Karur et al. (2013) discussed an optimal ordering policy for inventory model with non-instantaneous deteriorating items and stock dependent demand.

In this paper, an integrated single vendor and multiple buyer inventory system for deteriorating item is studied in which demand rate is decreasing function of time. Here we consider two type of buyers. First buyer assumes that the demand rate as stock dependent with two separate warehouses . When the stock level exceeds the capacity of the own warehouse, a rented warehouse is used. The stock is transformed periodically in bulk from RW to OW. The second buyer can return unsold items in a cycle time. The demand function is linear. The price discount is allowed in this model. Finally numerical example is provided.

2. Assumptions and notations

2.1 **Assumptions**

1. An inventory system of single vendor and multiple buyer is considered.
2. There is no replacement or repair of deteriorating items during the period under consideration.
3. The deteriorating items are non-instantaneous in first case.
4. The unsold items are returned after certain time.
5. The inventory cost in the RW are higher than those in the OW.
6. The goods of OW are consumed only after consuming the goods kept in RW.
7. OW has a fixed capacity W_{b1} units and RW has unlimited capacity.
8. Shortages are not allowed.
9. The lead time is zero.

2.2 **Notations**

The proposed model is derived using the following notations.

A_v-- Vendors ordering cost per unit.

A_{b1i} -- Buyers ordering cost per unit(case 1).

A_{b2i} -- Buyers ordering cost per unit(case 2).

$D(t)_{1i}$ –The demand rate which is linearly dependent on time, $a_{1i}(1-b_{1i}t)$ $a_{1i}> 0$, $0 < b_{1i}< 1$.

$D(t)_{2i}$ –The demand rate which is linearly dependent on time, $a_{2i}(1-b_{2i}t)$ $a_{2i}> 0$, $0 < b_{2i}< 1$.

h_v-- Vendors annual holding cost per time unit.

h_{b1i}-- Buyers annual holding cost per time unit.

h_{b2i} -- Buyers annual holding cost per time unit.

n -- Buyers order times during[0, T].

$I_v(t)$- Vendor inventory level.

$I_{b1i}(t)$- Buyers inventory level (case 1).

$I_{b2i}(t)$- Buyers inventory level (case 2).

θ -- The deterioration rate for vendor.

α -- The deterioration rate for buyer in case 1.

β -- The deterioration rate for buyer in case 2.

T—The length of the order cycle.

Q_v—The maximum inventory level of vendor.

Q_{b1i} —The maximum inventory level of buyer in case 1.
Q_{b2i} —The maximum inventory level of buyer in case 2.
DC_v –Vendors deteriorating cost per unit.
DC_{b1i}--Buyer deteriorating cost per unit in case 1.
DC_{b2i}-Buyer deteriorating cost per unit in case 2.
PC_v – Vendors purchasing cost per unit.
PC_{b1i}--Buyer purchasing cost per unit in case 1.
PC_{b2i}--Buyer purchasing cost per unit in case 2.
μ_1 –The time at which the inventory level reaches zero in RW in two warehouse system.
t_r --The unsold item are returned at the time.
TC_v –The total cost of Vendor per time unit.
TC_{b1i} - The total cost of all buyers per time unit in case 1.
TC_{b2i} -The total cost of all buyers per time unit in case 2.
TC –The total cost for vendor-buyers inventory system when they take decision jointly.

3. Mathematical Model

Let $I_{bi}(t)$ be the on hand inventory level for buyer i (i=1,2,……N) at any instant of time $0 \leq t \leq T_{bi}$ and let $I_v(t)$ is inventory level for vendor at any instant of time $0 \leq t \leq T$. As the inventory reduces due to demand rate as well as deterioration rate during the interval [0, T], the rate of change of inventory level is governed by the following differential equation for vendor and buyer.

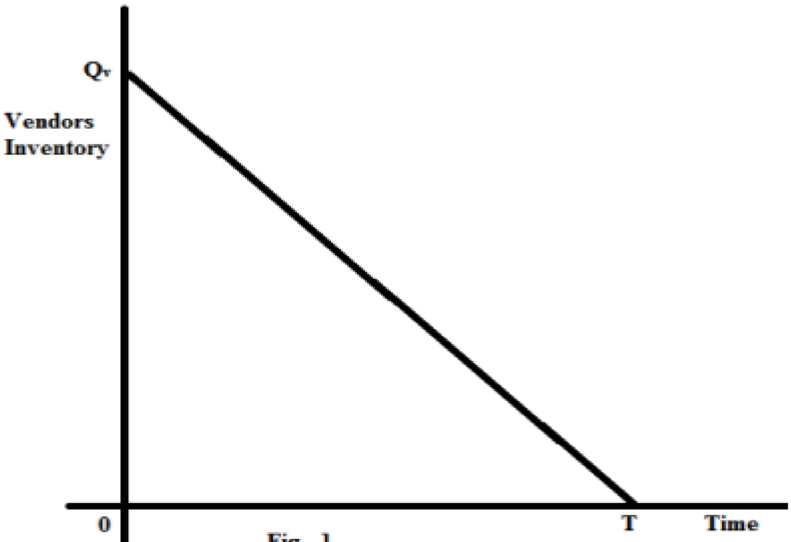

Fig 1

$$\frac{dI_v(t)}{dt} + \theta I_v(t) = -[D_{1i}(t) + D_{2i}(t)] \qquad 0 \leq t \leq T \qquad ------(1)$$

With the boundary conditions and initial conditions are
$I_v(0) = Q_v,\ I_v(T) = 0$

The solutions of the differential equation is

$$I_v(t) = (T-t)\sum_{i=0}^{N}[a_{1i}(1 - b_{1i}T) + a_{2i}(1 - b_{2i}T)] \ \ 0 \leq t \leq T \qquad ------(2)$$

$$Q_v = T\sum_{i=0}^{N}[a_{1i}(1-b_{1i}T)+a_{2i}(1-b_{2i}T)] \qquad ----(3)$$

The ordering cost is

$$OC_v = A_v \qquad ----(4)$$

The holding cost is

$$HC_v = h_v\left[\int_0^T I_v(t)dt - \sum_{i=0}^{N} n_{1i}\int_0^{T_{b1}} I_{b1i}(t)dt - \sum_{i=0}^{N} n_{2i}\int_0^{T_{b2}} I_{b2i}(t)dt\right] \qquad ----(5)$$

$$HC_v = h_v\left[\sum_{i=0}^{N}\frac{T^2}{2}\big(a_{1i}(1-b_{1i}T)+a_{2i}(1-b_{2i}T)\big)\right.$$
$$- h_{b1i}\sum n_{1i}\left[Q_{b1i}\mu_1 + \frac{W_{b1i}}{\alpha}(1-e^{-\alpha\mu_1}) + W_{b1i}\big(e^{\alpha(\mu_1-\mu_2)}-1\big)\right.$$
$$+\frac{a_{1i}{\mu_1}^2}{2}\left(\frac{\mu_1}{3}-1\right) - \frac{a_{1i}}{2}(\mu_1-\mu_2)^2 - a_{1i}b_{1i}\mu_1\mu_2\left(\mu_1-\frac{\mu_2}{2}\right) + \frac{a_{1i}b_{1i}{\mu_1}^3}{2}$$
$$+ a_{1i}\left(\frac{{T_{b1i}}^2}{2}(1+b_{1i}\mu_2) - \frac{b_{1i}}{3}\left(\frac{{\mu_2}^3}{2}+{T_{b1i}}^3\right) - T_{b1i}\mu_2 + \frac{{\mu_2}^2}{2} + \frac{b_{1i}\alpha {T_{b1i}}^5}{30}\right.$$
$$\left.\left. - \frac{\alpha {T_{b1i}}^4}{24} + \frac{b_{1i}\alpha{\mu_2}^3}{4}\left(\frac{{\mu_2}^2}{5}-\frac{{T_{b1i}}^2}{3}\right) + \frac{\alpha{\mu_2}^3}{2}\left(\frac{T_{b1i}}{3}-\frac{\mu_2}{4}\right)\right)\right]$$
$$- h_{b2i}\sum n_{2i}\left(\frac{Q_{b2i}}{\beta}(1-e^{-\beta t_r}) - a_{2i}{t_r}^2\left(3-\frac{b_{2i}t_r}{2}\right)\right.$$
$$\left.\left.+ a_{2i}T_{b2i}\left(t_r-\frac{T_{b2i}}{2}\right)\big((1-b_{2i}t_r)\big) + \frac{W_{b2i}}{\beta}\left(1-e^{\beta(t_r-T_{b2i})}\right)\right)\right]$$

The deteriorating cost is

$$DC_v = P_v\theta\left[\int_0^T I_v(t)dt - \sum_{i=0}^{N} n_{1i}\int_0^{T_{b1}} I_{b1i}(t)dt - \sum_{i=0}^{N} n_{2i}\int_0^{T_{b2}} I_{b2i}(t)dt\right] \qquad ----(6)$$

$$\mathrm{DC}_v = P_v\theta\left[\sum_{i=1}^{N}\frac{T^2}{2}\left(a_{1i}(1-b_{1i}\mathrm{T})+a_{2i}(1-b_{2i}\mathrm{T})\right)\right.$$
$$-\mathrm{P_{b1i}}\alpha\sum \mathrm{n_{1i}}\left[\frac{\mathrm{W_{b1i}}}{\alpha}(1-e^{-\alpha\mu_1})+\mathrm{W_{b1i}}\left(e^{\alpha(\mu_1-\mu_2)}-1\right)-\frac{\mathrm{a_{1i}}}{2}(\mu_1-\mu_2)^2\right.$$
$$-\mathrm{a_{1i}b_{1i}}\mu_1\mu_2\left(\mu_1-\frac{\mu_2}{2}\right)+\frac{\mathrm{a_{1i}b_{1i}}{\mu_1}^3}{2}$$
$$+\mathrm{a_{1i}}\left(\frac{{\mathrm{T_{b1i}}}^3}{6}-\frac{\alpha{\mathrm{T_{b1i}}}^5}{40}-\frac{\mathrm{b_{1i}}{\mathrm{T_{b1i}}}^4}{8}+\frac{\mathrm{b_{1i}}\alpha{\mathrm{T_{b1i}}}^6}{48}+{\mu_2}^2\left(\frac{\mu_2}{3}-\frac{\mathrm{T_{b1i}}}{2}\right)\right.$$
$$\left.\left.+\frac{\alpha{\mu_2}^4}{2}\left(\frac{\mathrm{T_{b1i}}}{4}-\frac{\mu_2}{5}\right)+\frac{\mathrm{b_{1i}}{\mu_2}^2}{4}\left({\mathrm{T_{b1i}}}^2-\frac{{\mu_2}^2}{2}\right)-\frac{\mathrm{b_{1i}}\alpha{\mu_2}^4}{4}\left(\frac{{\mathrm{T_{b1i}}}^2}{4}-\frac{{\mu_2}^2}{6}\right)\right)\right]$$
$$-\beta\mathrm{P_{b2i}}\sum \mathrm{n_{2i}}\left(\frac{\mathrm{Q_{b2i}}}{\beta}(1-e^{-\beta t_r})-\mathrm{a_{2i}}{t_r}^2\left(3-\frac{b_{2i}\mathrm{t_r}}{2}\right)\right.$$
$$\left.\left.+\mathrm{a_{2i}}\mathrm{T}_{b2i}\left(t_r-\frac{\mathrm{T}_{b2i}}{2}\right)\left((1-b_{2i}\mathrm{t_r})\right)+\frac{W_{b2i}}{\beta}\left(1-e^{\beta(t_r-\mathrm{T}_{b2i})}\right)\right)\right]$$

The purchasing cost is

$$\mathrm{PC}_v = P_v * Q_v - \sum \mathrm{n_{1i}\,Q_{b1i}(P_{b1i}-0.2P_{b1i})} - \sum \mathrm{n_{2i}\,Q_{b2i}P_{b2i}} \quad ----- (7)$$

Based on order quantity and predetermined order quantity we have the following two cases

1) $Q_{b1i} > Q_f$ 2) $Q_{b2i} < Q_f$

3.1 Case: I$Q_{b1i} > Q_f$

Here the deteriorating items are available, but different deterioration in the cycle time (0, T). At time t=0 a lot size of Q_{b1i}unit enters the system.During the inventory [0, μ_1], the inventory level are positive at RW and OW are decreasing only owing to stock dependent demand rate.In RW, during (μ_1, T) the inventory is depleted due to both demand and deterioration, until it reaches zero level at time t=T. During the interval (μ_1, μ_2) inventory depletes due to deterioration at rate α. During the interval (μ_2, T) inventory depletes due to deterioration at the rate of αt. The differential equation which describes the instantaneous states of I_{b1i} (t) in RW and OW respectively, over the period (0, T) are given by

$$\frac{\mathrm{dI_{b1i}(t)}}{\mathrm{dt}} = -\mathrm{D_{1i}(t)} \qquad 0 \le \mathrm{t} \le \mu_1 \qquad ------ (8)$$

$$\frac{\mathrm{dI_{b1i}(t)}}{\mathrm{dt}} + \alpha\mathrm{I_{b1i}(t)} = 0 \qquad 0 \le \mathrm{t} \le \mu_1 \qquad ------ (9)$$

$$\frac{\mathrm{dI_{b1i}(t)}}{\mathrm{dt}} + \alpha\mathrm{I_{b1i}(t)} = -\mathrm{D_{1i}(t)}\mu_1 \le \mathrm{t} \le \mu_2 \qquad ------ (10)$$

$$\frac{\mathrm{dI_{b1i}(t)}}{\mathrm{dt}} + \alpha\mathrm{tI_{b1i}(t)} = -\mathrm{D_{1i}(t)}\mu_2 \le \mathrm{t} \le \mathrm{T_{b1i}} \qquad ------ (11)$$

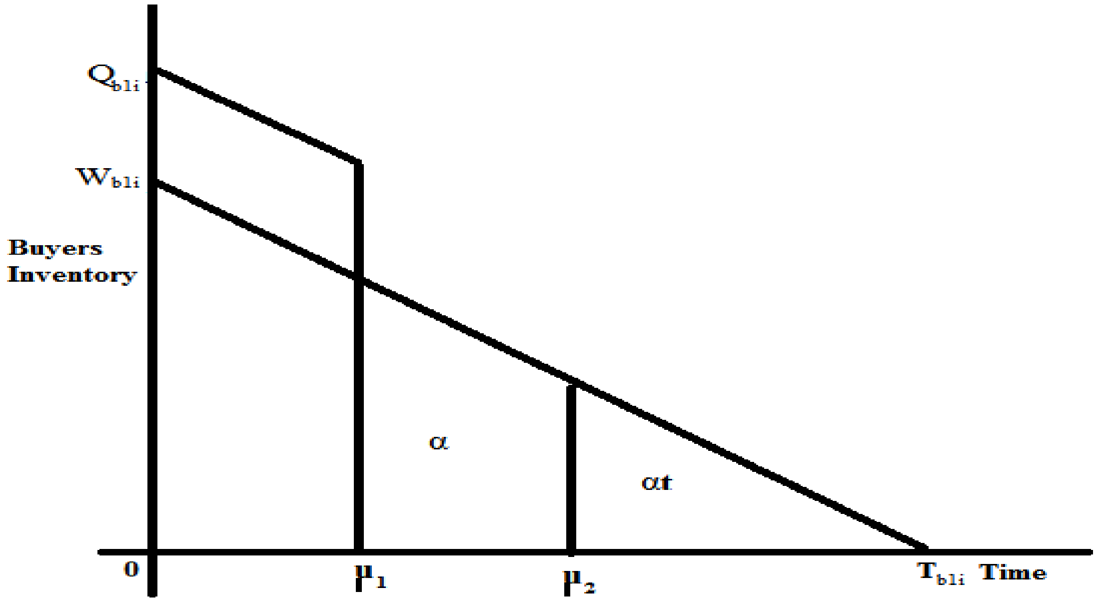

Fig 2

With initial conditions $I_{b1i}(0) = Q_{b1i}, I_{b1i}(0) = W_{b1i}, I_{b1i}(\mu_1) = \alpha W_{b1i}, I_{b1i}(T_{b1i}) = 0$ solutions of these equations are given by

$$I_{b1i}(t) = -a_{1i}\left(t - b_{1i}\frac{t^2}{2}\right) + Q_{b1i} \qquad 0 \le t \le \mu_1 \qquad ------(12)$$

$$I_{b1i}(t) = W_{b1i}e^{-\alpha t} \qquad 0 \le t \le \mu_1 \qquad ------(13)$$

$$I_{b1i}(t) = [a_{1i}(\mu_1 - t)(1 - b_{1i}\mu_1)] + \alpha W_{b1i}e^{\alpha(\mu_1 - t)}\mu_1 \le t \le \mu_2 \qquad ------(14)$$

$$I_{b1i}(t) = a_{1i}\left[(T_{b1i} - t) - \frac{b_{1i}}{2}({T_{b1i}}^2 - t^2)\right]e^{-\alpha\frac{t^2}{2}}\mu_2 \le t \le T_{b1i} \qquad ------(15)$$

$$Q_{b1i} = W_{b1i} + a_{1i}\left[\mu_1 - \frac{b_{1i}{\mu_1}^2}{2}\right] \qquad -----(16)$$

The ordering cost is $OC_{b1i} = \sum_{i=1}^{N} n_{1i}A_{bi}$ ------ (17)

The holding cost is $HC_{b1i} = h_{b1i}\sum n_{1i}\int_0^{T_{b1i}} I_{b1i}(t)dt$ ------ (18)

$$HC_{b1i} = h_{b1i}\sum n_{1i}\Bigg[Q_{b1i}\mu_1 + \frac{W_{b1i}}{\alpha}(1 - e^{-\alpha\mu_1}) + W_{b1i}\left(e^{\alpha(\mu_1-\mu_2)} - 1\right) + \frac{a_{1i}{\mu_1}^2}{2}\left(\frac{\mu_1}{3} - 1\right) - \frac{a_{1i}}{2}(\mu_1 - \mu_2)^2 - a_{1i}b_{1i}\mu_1\mu_2\left(\mu_1 - \frac{\mu_2}{2}\right) + \frac{a_{1i}b_{1i}{\mu_1}^3}{2} + a_{1i}\left(\frac{{T_{b1i}}^2}{2}(1 + b_{1i}\mu_2) - \frac{b_{1i}}{3}\left(\frac{{\mu_2}^3}{2} + {T_{b1i}}^3\right) - T_{b1i}\mu_2 + \frac{{\mu_2}^2}{2} + \frac{b_{1i}\alpha {T_{b1i}}^5}{30} - \frac{\alpha {T_{b1i}}^4}{24} + \frac{b_{1i}\alpha{\mu_2}^3}{4}\left(\frac{{\mu_2}^2}{5} - \frac{{T_{b1i}}^2}{3}\right) + \frac{\alpha{\mu_2}^3}{2}\left(\frac{T_{b1i}}{3} - \frac{\mu_2}{4}\right)\right)\Bigg]$$

The deteriorating cost is

$$DC_{b1i} = P_{b1i}\sum n_{1i}\left[\int_0^{\mu_1}\alpha I_{b1i}(t)dt + \int_{\mu_1}^{\mu_2}\alpha I_{b1i}(t)dt + \int_{\mu_2}^{T_{b1i}}\alpha t I_{b1i}(t)dt\right] \quad ----- (19)$$

$$DC_{b1i} = P_{b1i}\alpha\sum n_{1i}\left[\frac{W_{b1i}}{\alpha}(1-e^{-\alpha\mu_1}) + W_{b1i}\left(e^{\alpha(\mu_1-\mu_2)}-1\right) - \frac{a_{1i}}{2}(\mu_1-\mu_2)^2 - a_{1i}b_{1i}\mu_1\mu_2\left(\mu_1-\frac{\mu_2}{2}\right) + \frac{a_{1i}b_{1i}\mu_1^{\,3}}{2} + a_{1i}\left(\frac{T_{b1i}^{\,3}}{6} - \frac{\alpha T_{b1i}^{\,5}}{40} - \frac{b_{1i}T_{b1i}^{\,4}}{8} + \frac{b_{1i}\alpha T_{b1i}^{\,6}}{48} + \mu_2^{\,2}\left(\frac{\mu_2}{3}-\frac{T_{b1i}}{2}\right) + \frac{\alpha\mu_2^{\,4}}{2}\left(\frac{T_{b1i}}{4}-\frac{\mu_2}{5}\right) + \frac{b_{1i}\mu_2^{\,2}}{4}\left(T_{b1i}^{\,2}-\frac{\mu_2^{\,2}}{2}\right) - \frac{b_{1i}\alpha\mu_2^{\,4}}{4}\left(\frac{T_{b1i}^{\,2}}{4}-\frac{\mu_2^{\,2}}{6}\right)\right)\right]$$

The purchase cost is

$$PC_{b1i} = \sum n_{1i}\, Q_{b1i}(P_{b1i} - 0.2P_{b1i}) \quad ----- (20)$$

3.2 Case:II $\quad Q_{b2i} < Q_f$

The initial inventory level is Q_{b2i}unit at time t=0. From t=0 to t=T the inventory level reduces, owing to both demand and deterioration.At time t=t_r, the unsold items are returned, again inventory level deceases due to demand and deterioration until it reaches zero level at timet=T. The rate of change of the inventory during the positive stock period (0, T) is governed by the following differential equations:

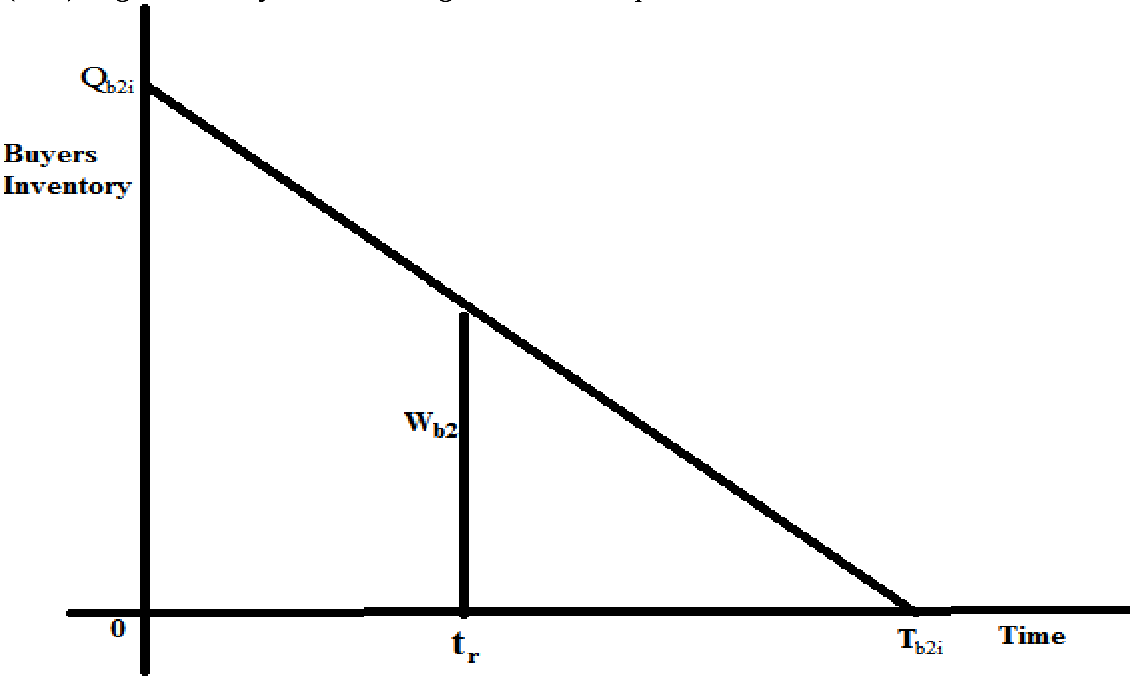

Fig 3

$$\frac{dI_{b2i}(t)}{dt} + \beta I_{b2i}(t) = -D_{2i}(t) \qquad 0 \le t \le t_r \quad ----- (21)$$

$$\frac{dI_{b2i}(t)}{dt} + \beta I_{b2i}(t) = -D_{2i}(t) t_r \le t \le T_{b2i} \qquad ------(22)$$

Thus, boundary conditions are a follows

$$I_{b2i}(0) = Q_{b2i}, I_{b2i}(t_r) = W_{b2i}, I_b(T_{b2i}) = 0$$

The solution of equations ---- and -----with boundary conditions as follows

$$I_{b2i}(t) = -a_{2i}t + Q_{b2i}e^{-\beta t} \qquad 0 \le t \le t_r \qquad ------(23)$$

$$I_{b2i}(t) = a_{2i}(t_r - t)(1 - b_{2i}t_r) + W_{b2i}e^{\beta(t_r - t)} t_r \le t \le T_{b2i} \qquad ------(24)$$

$$W_{b2i} = [a_{2i}(T_{b2i} - t_r)(1 - b_{2i}t_r)]e^{\beta(T_{b2} - t_r)} \qquad -----(25)$$

$$Q_{b2i} = [a_{2i}(T_{b2i} - t_r)(1 - b_{2i}t_r)]e^{\beta T_{b2i}} + a_{2i}t_r e^{\beta t_r} \qquad -----(26)$$

The ordering cost is

$$OC_{b2i} = \sum n_{2i}A_{b2i} \qquad -----(27)$$

The holding cost is

$$HC_{b2i} = h_{b2i}\sum n_{2i}\int_0^{T_{b2i}} I_{b2i}(t)dt \qquad -----(28)$$

$$HC_{b2i} = h_{b2i}\sum n_{2i}\left(\frac{Q_{b2i}}{\beta}\left(1 - e^{-\beta t_r}\right) - a_{2i}{t_r}^2\left(3 - \frac{b_{2i}t_r}{2}\right) + a_{2i}T_{b2i}\left(t_r - \frac{T_{b2i}}{2}\right)\left((1 - b_{2i}t_r)\right) + \frac{W_{b2i}}{\beta}\left(1 - e^{\beta(t_r - T_{b2i})}\right)\right)$$

The deterioration cost is

$$DC_{b2i} = P_{b2i}\sum n_{2i}\left[\int_0^{T_{b2i}} \beta I_{b2i}(t)dt\right] \qquad -----(29)$$

$$DC_{b2i} = \beta P_{b2i}\sum n_{2i}\left(\frac{Q_{b2i}}{\beta}\left(1 - e^{-\beta t_r}\right) - a_{2i}{t_r}^2\left(3 - \frac{b_{2i}t_r}{2}\right) + a_{2i}T_{b2i}\left(t_r - \frac{T_{b2i}}{2}\right)\left((1 - b_{2i}t_r)\right) + \frac{W_{b2i}}{\beta}\left(1 - e^{\beta(t_r - T_{b2i})}\right)\right)$$

The purchase cost is

$$PC_{b2i} = \sum n_{2i}\, Q_{b2i}P_{b2i} \qquad -----(30)$$

Hence, the vendors total cost TC_v per unit time is

$$TC_v = \frac{1}{T}[OC_V + HC_V + DC_V + PC_V] \qquad -----(31)$$

$$
\begin{aligned}
\mathrm{TC}_v = \frac{1}{T}\Bigg[& A_v + h_v \Bigg[\sum_{i=0}^{N} \frac{T^2}{2}\big(a_{1i}(1-b_{1i}\mathrm{T}) + a_{2i}(1-b_{2i}\mathrm{T})\big) \\
& - \mathrm{h}_{\mathrm{b1i}} \sum \mathrm{n}_{1\mathrm{i}} \Bigg[\mathrm{Q}_{\mathrm{b1i}}\mu_1 + \frac{\mathrm{W}_{\mathrm{b1i}}}{\alpha}\left(1-e^{-\alpha\mu_1}\right) + \mathrm{W}_{\mathrm{b1i}}\left(e^{\alpha(\mu_1-\mu_2)}-1\right) \\
& + \frac{\mathrm{a}_{1\mathrm{i}}{\mu_1}^2}{2}\left(\frac{\mu_1}{3}-1\right) - \frac{\mathrm{a}_{1\mathrm{i}}}{2}(\mu_1-\mu_2)^2 - \mathrm{a}_{1\mathrm{i}}\mathrm{b}_{1\mathrm{i}}\mu_1\mu_2\left(\mu_1-\frac{\mu_2}{2}\right) + \frac{\mathrm{a}_{1\mathrm{i}}\mathrm{b}_{1\mathrm{i}}{\mu_1}^3}{2} \\
& + \mathrm{a}_{1\mathrm{i}}\Bigg(\frac{{\mathrm{T}_{\mathrm{b1i}}}^2}{2}(1+\mathrm{b}_{1\mathrm{i}}\mu_2) - \frac{\mathrm{b}_{1\mathrm{i}}}{3}\left(\frac{{\mu_2}^3}{2}+{\mathrm{T}_{\mathrm{b1i}}}^3\right) - \mathrm{T}_{\mathrm{b1i}}\mu_2 + \frac{{\mu_2}^2}{2} + \frac{\mathrm{b}_{1\mathrm{i}}\alpha{\mathrm{T}_{\mathrm{b1i}}}^5}{30} \\
& - \frac{\alpha{\mathrm{T}_{\mathrm{b1i}}}^4}{24} + \frac{\mathrm{b}_{1\mathrm{i}}\alpha{\mu_2}^3}{4}\left(\frac{{\mu_2}^2}{5}-\frac{{\mathrm{T}_{\mathrm{b1i}}}^2}{3}\right) + \frac{\alpha{\mu_2}^3}{2}\left(\frac{\mathrm{T}_{\mathrm{b1i}}}{3}-\frac{\mu_2}{4}\right)\Bigg)\Bigg] \\
& - \mathrm{h}_{\mathrm{b2i}}\sum \mathrm{n}_{2\mathrm{i}}\Bigg(\frac{\mathrm{Q}_{\mathrm{b2i}}}{\beta}\left(1-e^{-\beta t_r}\right) - \mathrm{a}_{2\mathrm{i}}{t_r}^2\left(3-\frac{b_{2i}\mathrm{t_r}}{2}\right) \\
& + \mathrm{a}_{2\mathrm{i}}\mathrm{T}_{b2i}\left(t_r-\frac{\mathrm{T}_{b2i}}{2}\right)\big((1-b_{2i}\mathrm{t_r})\big) + \frac{W_{b2i}}{\beta}\left(1-e^{\beta(t_r-\mathrm{T}_{b2i})}\right)\Bigg)\Bigg] \\
& + P_v\theta\Bigg[\sum_{i=1}^{N}\frac{T^2}{2}\big(a_{1i}(1-b_{1i}\mathrm{T}) + a_{2i}(1-b_{2i}\mathrm{T})\big) \\
& - \mathrm{P}_{\mathrm{b1i}}\alpha\sum \mathrm{n}_{1\mathrm{i}}\Bigg[\frac{\mathrm{W}_{\mathrm{b1i}}}{\alpha}\left(1-e^{-\alpha\mu_1}\right) + \mathrm{W}_{\mathrm{b1i}}\left(e^{\alpha(\mu_1-\mu_2)}-1\right) - \frac{\mathrm{a}_{1\mathrm{i}}}{2}(\mu_1-\mu_2)^2 \\
& - \mathrm{a}_{1\mathrm{i}}\mathrm{b}_{1\mathrm{i}}\mu_1\mu_2\left(\mu_1-\frac{\mu_2}{2}\right) + \frac{\mathrm{a}_{1\mathrm{i}}\mathrm{b}_{1\mathrm{i}}{\mu_1}^3}{2} \\
& + \mathrm{a}_{1\mathrm{i}}\Bigg(\frac{{\mathrm{T}_{\mathrm{b1i}}}^3}{6} - \frac{\alpha{\mathrm{T}_{\mathrm{b1i}}}^5}{40} - \frac{\mathrm{b}_{1\mathrm{i}}{\mathrm{T}_{\mathrm{b1i}}}^4}{8} + \frac{\mathrm{b}_{1\mathrm{i}}\alpha{\mathrm{T}_{\mathrm{b1i}}}^6}{48} + {\mu_2}^2\left(\frac{\mu_2}{3}-\frac{\mathrm{T}_{\mathrm{b1i}}}{2}\right) \\
& + \frac{\alpha{\mu_2}^4}{2}\left(\frac{\mathrm{T}_{\mathrm{b1i}}}{4}-\frac{\mu_2}{5}\right) + \frac{\mathrm{b}_{1\mathrm{i}}{\mu_2}^2}{4}\left({\mathrm{T}_{\mathrm{b1i}}}^2-\frac{{\mu_2}^2}{2}\right) - \frac{\mathrm{b}_{1\mathrm{i}}\alpha{\mu_2}^4}{4}\left(\frac{{\mathrm{T}_{\mathrm{b1i}}}^2}{4}-\frac{{\mu_2}^2}{6}\right)\Bigg)\Bigg] \\
& - \beta\mathrm{P}_{\mathrm{b2i}}\sum \mathrm{n}_{2\mathrm{i}}\Bigg(\frac{\mathrm{Q}_{\mathrm{b2i}}}{\beta}\left(1-e^{-\beta t_r}\right) - \mathrm{a}_{2\mathrm{i}}{t_r}^2\left(3-\frac{b_{2i}\mathrm{t_r}}{2}\right) \\
& + \mathrm{a}_{2\mathrm{i}}\mathrm{T}_{b2i}\left(t_r-\frac{\mathrm{T}_{b2i}}{2}\right)\big((1-b_{2i}\mathrm{t_r})\big) + \frac{W_{b2i}}{\beta}\left(1-e^{\beta(t_r-\mathrm{T}_{b2i})}\right)\Bigg)\Bigg] + P_v * Q_v \\
& - \sum \mathrm{n}_{1\mathrm{i}}\,\mathrm{Q}_{\mathrm{b1i}}(\mathrm{P}_{\mathrm{b1i}} - 0.2\mathrm{P}_{\mathrm{b1i}}) - \sum \mathrm{n}_{2\mathrm{i}}\,\mathrm{Q}_{\mathrm{b2i}}\mathrm{P}_{\mathrm{b2i}}\Bigg]
\end{aligned}
$$

The Buyers total cost $TC_{b1i}$and TC_{b2i}per unit time is

$$
\mathrm{TC}_{\mathrm{b1i}} = \frac{1}{\mathrm{T}_{b1i}}[\mathrm{OC}_{\mathrm{b1i}} + \mathrm{HC}_{\mathrm{b1i}} + \mathrm{DC}_{\mathrm{b1i}} + \mathrm{PC}_{\mathrm{b1i}}] \qquad ----(31)
$$

$$TC_{b2i} = \frac{1}{T_{b2i}}\left[OC_{b2i} + HC_{b2i} + DC_{b2i} + PC_{b2i}\right] \quad ----- (32)$$

$$\begin{aligned}
TC_{b1i} = \frac{1}{T_{b1i}}\Bigg[&\sum_{i=1}^{N} n_{1i}A_{b1i} \\
&+ h_{b1i}\sum n_{1i}\Bigg[Q_{b1i}\mu_1 + \frac{W_{b1i}}{\alpha}\left(1 - e^{-\alpha\mu_1}\right) + W_{b1i}\left(e^{\alpha(\mu_1-\mu_2)} - 1\right) \\
&+ \frac{a_{1i}{\mu_1}^2}{2}\left(\frac{\mu_1}{3} - 1\right) - \frac{a_{1i}}{2}(\mu_1 - \mu_2)^2 - a_{1i}b_{1i}\mu_1\mu_2\left(\mu_1 - \frac{\mu_2}{2}\right) + \frac{a_{1i}b_{1i}{\mu_1}^3}{2} \\
&+ a_{1i}\Bigg(\frac{{T_{b1i}}^2}{2}(1 + b_{1i}\mu_2) - \frac{b_{1i}}{3}\left(\frac{{\mu_2}^3}{2} + {T_{b1i}}^3\right) - T_{b1i}\mu_2 + \frac{{\mu_2}^2}{2} + \frac{b_{1i}\alpha {T_{b1i}}^5}{30} \\
&- \frac{\alpha {T_{b1i}}^4}{24} + \frac{b_{1i}\alpha{\mu_2}^3}{4}\left(\frac{{\mu_2}^2}{5} - \frac{{T_{b1i}}^2}{3}\right) + \frac{\alpha{\mu_2}^3}{2}\left(\frac{T_{b1i}}{3} - \frac{\mu_2}{4}\right)\Bigg)\Bigg] \\
&+ P_{b1i}\alpha\sum n_{1i}\Bigg[\frac{W_{b1i}}{\alpha}\left(1 - e^{-\alpha\mu_1}\right) + W_{b1i}\left(e^{\alpha(\mu_1-\mu_2)} - 1\right) - \frac{a_{1i}}{2}(\mu_1 - \mu_2)^2 \\
&- a_{1i}b_{1i}\mu_1\mu_2\left(\mu_1 - \frac{\mu_2}{2}\right) + \frac{a_{1i}b_{1i}{\mu_1}^3}{2} \\
&+ a_{1i}\Bigg(\frac{{T_{b1i}}^3}{6} - \frac{\alpha{T_{b1i}}^5}{40} - \frac{b_{1i}{T_{b1i}}^4}{8} + \frac{b_{1i}\alpha{T_{b1i}}^6}{48} + {\mu_2}^2\left(\frac{\mu_2}{3} - \frac{T_{b1i}}{2}\right) \\
&+ \frac{\alpha{\mu_2}^4}{2}\left(\frac{T_{b1i}}{4} - \frac{\mu_2}{5}\right) + \frac{b_{1i}{\mu_2}^2}{4}\left({T_{b1i}}^2 - \frac{{\mu_2}^2}{2}\right) - \frac{b_{1i}\alpha{\mu_2}^4}{4}\left(\frac{{T_{b1i}}^2}{4} - \frac{{\mu_2}^2}{6}\right)\Bigg)\Bigg] \\
&+ \sum n_{1i}\,Q_{b1i}(P_{b1i} - 0.2P_{b1i})\Bigg]
\end{aligned}$$

$$\begin{aligned}
TC_{b2i} = \frac{1}{T_{b2i}}\Bigg[&\sum n_{2i}A_{b2i} \\
&+ h_{b2i}\sum n_{2i}\Bigg(\frac{Q_{b2i}}{\beta}\left(1 - e^{-\beta t_r}\right) - a_{2i}{t_r}^2\left(3 - \frac{b_{2i}t_r}{2}\right) \\
&+ a_{2i}T_{b2i}\left(t_r - \frac{T_{b2i}}{2}\right)\left((1 - b_{2i}t_r)\right) + \frac{W_{b2i}}{\beta}\left(1 - e^{\beta(t_r - T_{b2i})}\right)\Bigg) \\
&+ \beta P_{b2i}\sum n_{2i}\Bigg(\frac{Q_{b2i}}{\beta}\left(1 - e^{-\beta t_r}\right) - a_{2i}{t_r}^2\left(3 - \frac{b_{2i}t_r}{2}\right) \\
&+ a_{2i}T_{b2i}\left(t_r - \frac{T_{b2i}}{2}\right)\left((1 - b_{2i}t_r)\right) + \frac{W_{b2i}}{\beta}\left(1 - e^{\beta(t_r - T_{b2i})}\right)\Bigg) \\
&+ \sum n_{2i}\,Q_{b2i}P_{b2i}\Bigg]
\end{aligned}$$

The optimum value of cycle time T_{b1i}, T_{b2i}, n_{1i} and n_{2i} can be obtained by following necessary condition.

$$\frac{dTC_{b1i}}{dT_{b1i}} = 0, \frac{dTC_{b2i}}{dT_{b2i}} = 0 \qquad -----(33)$$

$$\frac{dTC_{b1i}}{dT_{b1i}} = -\frac{1}{{T_{b1i}}^2}\Bigg[\sum_{i=1}^{N} n_{1i}A_{b1i} + h_{b1i}\sum n_{1i}\Bigg(Q_{b1i}\mu_1 + \frac{W_{b1i}}{\alpha}(1-e^{-\alpha\mu_1}) + W_{b1i}(e^{\alpha(\mu_1-\mu_2)}-1) + \frac{a_{1i}{\mu_1}^2}{2}\left(\frac{\mu_1}{3}-1\right) - \frac{a_{1i}}{2}(\mu_1-\mu_2)^2 - a_{1i}b_{1i}\mu_1\mu_2\left(\mu_1-\frac{\mu_2}{2}\right) + \frac{a_{1i}b_{1i}{\mu_1}^3}{2} - \frac{a_{1i}b_{1i}{\mu_2}^3}{3} + \frac{a_{1i}{\mu_2}^2}{2} + \frac{a_{1i}b_{1i}\alpha{\mu_2}^5}{20} - \frac{a_{1i}\alpha{\mu_2}^4}{8}\Bigg) + P_{b1i}\alpha\sum n_{1i}\Bigg(\frac{W_{b1i}}{\alpha}(1-e^{-\alpha\mu_1}) + W_{b1i}(e^{\alpha(\mu_1-\mu_2)}-1) - \frac{a_{1i}}{2}(\mu_1-\mu_2)^2 - a_{1i}b_{1i}\mu_1\mu_2\left(\mu_1-\frac{\mu_2}{2}\right) + \frac{a_{1i}b_{1i}{\mu_1}^3}{2} + \frac{a_{1i}{\mu_2}^3}{3} + \frac{a_{1i}\alpha{\mu_2}^5}{10} - \frac{a_{1i}b_{1i}{\mu_2}^4}{8} + \frac{a_{1i}b_{1i}\alpha{\mu_2}^6}{24}\Bigg) + \sum n_{1i}\,Q_{b1i}(P_{b1i}-0.2P_{b1i})\Bigg] + h_{b1i}\sum n_{1i}a_{1i}\left(\frac{(1+b_{1i}\mu_2)}{2} - \frac{T_{b1i}b_{1i}}{3} + \frac{2b_{1i}\alpha{T_{b1i}}^3}{15} - \frac{b_{1i}\alpha{\mu_2}^3}{12}\right) + P_{b1i}\alpha\sum n_{1i}a_{1i}\left(\frac{T_{b1i}}{3} - \frac{\alpha{T_{b1i}}^3}{10} - \frac{3b_{1i}{T_{b1i}}^2}{8} + \frac{5b_{1i}\alpha{T_{b1i}}^4}{48} + \frac{b_{1i}{\mu_2}^2}{4} - \frac{b_{1i}\alpha{\mu_2}^4}{16}\right)$$

$$\frac{dTC_{b2i}}{dt} = -\frac{1}{{T_{b2i}}^2}\Bigg[\sum n_{2i}A_{b2i} + \sum n_{2i}(h_{b2i}+\beta P_{b2i})\left(\frac{Q_{b2i}}{\beta}(1-e^{-\beta t_r}) - a_{2i}{t_r}^2\left(3-\frac{b_{2i}t_r}{2}\right) + \frac{W_{b2i}}{\beta}\right) + \sum n_{2i}\,Q_{b2i}P_{b2i}\Bigg] + \sum n_{2i}(h_{b2i}+\beta P_{b2i})\left(\frac{W_{b2i}}{\beta}(1-e^{\beta(t_r-T_{b2i})}) - \frac{a_{2i}}{2}(1-b_{2i}t_r)\right)$$

And TC (n-1) ≥ TC (n) ≥ TC (n+1), $\qquad -----(34)$

The total cost considering joint decision TC is

TC=Min (TC_{b1i}+TC_{b2i}+TC_v), where $TC_{b1i} = \frac{T}{n_{1i}}, TC_{b2i} = \frac{T}{n_{2i}}$

4. Numerical Example

Following numerical example will illustrate the model

The values of the various parameters in proper units can be taken as follows:

a_{1i}=450, a_{2i}=800, b_{1i}=0.5, b_{2i}=0.08, μ_1=0.10, μ_2=0.15, A_v=1000, A_{b1i}=200, A_{b2i}=200, t_r=0.085, α=0.15, β=0.08, θ=0.1, h_v=0.15, h_{b1i}=0.50, h_{b2i}=0.80, P_v=10, P_{b1i}=13, P_{b2i}=14.

The optimal value of n_i=3, T=0.90, and TC=84414.42.

5. Conclusion

In this paper, we have developed a single vendor and multiple buyer's inventory model for single items.Unequal sized shipment under deterministic demand, and different deterioration rate of units in the vendor buyer inventory system is considered. The joint total cost of the vendor-buyer is derived and a solution procedure is provided to find the optimal solution that can minimize the joint total cost.

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